Name: Sebastien Corner Date: 4/17/2013

**Task 1a. Excitation Selection [6 pts]**

Provide a detailed description of the excitation you chose for System Identification?

- The excitation we chose for this this system identification is two white noise signal, made with the randn() function with a mean of 0 and standard deviation of 3.2 . To avoid any DC gain we filtered this signal using a High pass filter with a cut off frequency of 0.03 Hz. We also added a Low pass filter with a cut off frequency of 1 Hz in order to reduce by few dB the amplitude of the second mode. We saw also the magnitude of the outputs were dropping by 40dB by decade after the second mode, therefore, it was not necessary to have an input signal that excite the system at the range of frequency [2Hz-10Hz]. Cutting off the excitation by the high pass filter allowed us to increase the standard of deviation, thus increased the standard of deviation of the output also and provide a higher signal ratio.
- Other excitation were texted such that the chirp excitation. Since the spectrum of this excitation is not flat ( between [0.07 1Hz]) as the white noise is, this excitation does not give a good result. We tried also to excite the system at the resonance frequency to see the magnitude of the output signal. This should not be tested in real application but only in simulation.
- For information we chose a number of sample N=5*nfft, with nfft=$2^{11}$.This give a time delay of 8 min and 31 sec for data acquisition ( real time).

Why is this particular excitation signal adequate for this plant?

- The white noise signal has the advantage of having a flat magnitude at the frequency range of interest. Therefore it excites the system at every frequency with constant magnitudes that leads to a better estimation of the mode localization.
- White noises have also the advantage of being uncorrelated which is important when we have to deal with MIMO system plant. To avoid any correlation between inputs we decided also to look at the response using a Chanel by Chanel excitation.

**Task 1b. Time- and Frequency-Domain Excitation [10 pts]**

Below is figure 1 that shows the time domain signal and spectrum of both excitations u1 and u2. As shown the amplitude of both excitation do not cross the provided limitation.

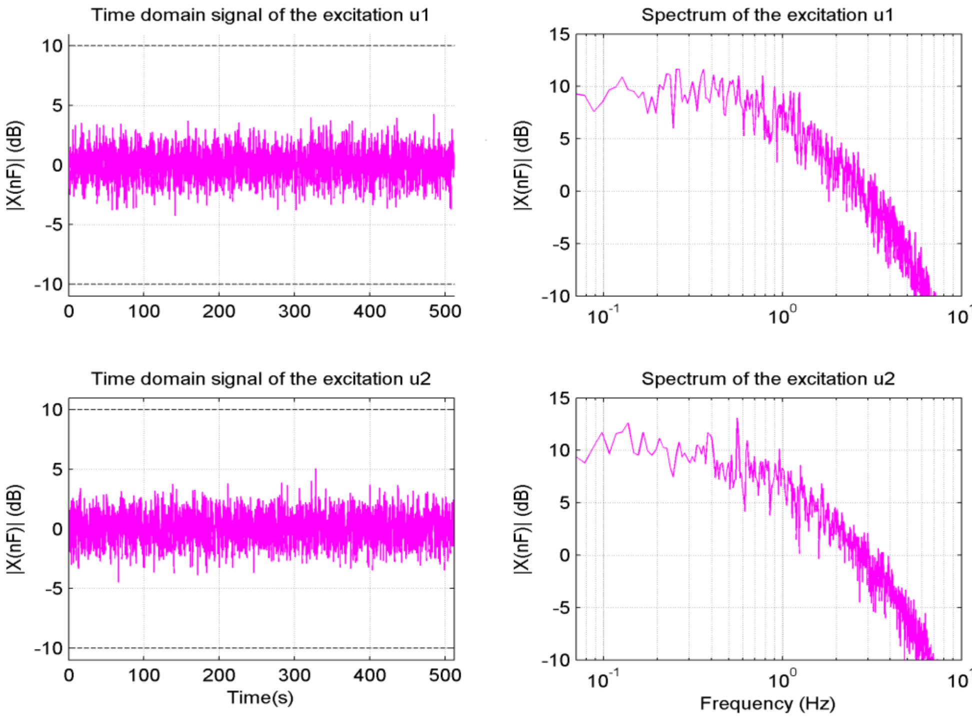


Figure 1 time domain signal and spectrum of the excitation

### Task 1c. SNR Estimation [8 pts]

The estimated SNR (dB) during System Identification was:

| | $u_1$ | $u_2$ |
|---|---|---|
| $y_1$ | 38.84 | 31.17 |
| $y_2$ | 32.96 | 31.407 |
| $y_3$ | 29.41 | 36.78 |

Explain how the SNR's were estimated:

- The SNR's were estimated by calculating the difference between the power of the signal output of the chosen channel and the power of the noise divided by the power of the noise. The noise power were estimated by calculating the square value of the standard of deviation of the output signal when both input were not activated (signal of magnitude zeros). The signal output power were estimated by calculating the square value of the standard of deviation of the output signal with one active excitation only, other input was not active (Chanel by Chanel excitation).

### Task 1d. Time- and Frequency-Domain Responses [20 pts]

Bellow are figure 2 and figure3 that show the time domain signal and spectrum of the sensor response y1, y2 and y3 for input u1 and u2 activated once at a time respectively. As shown the sensor response in not saturated

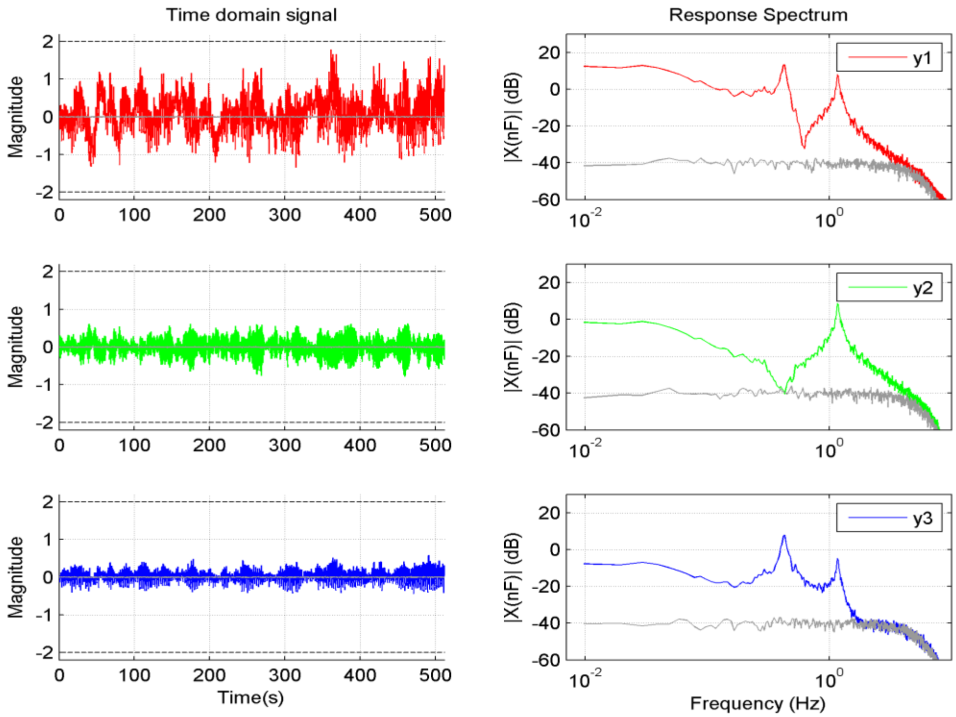


**Figure 2 time domain signal and spectrum of the sensor response for u1 activated**

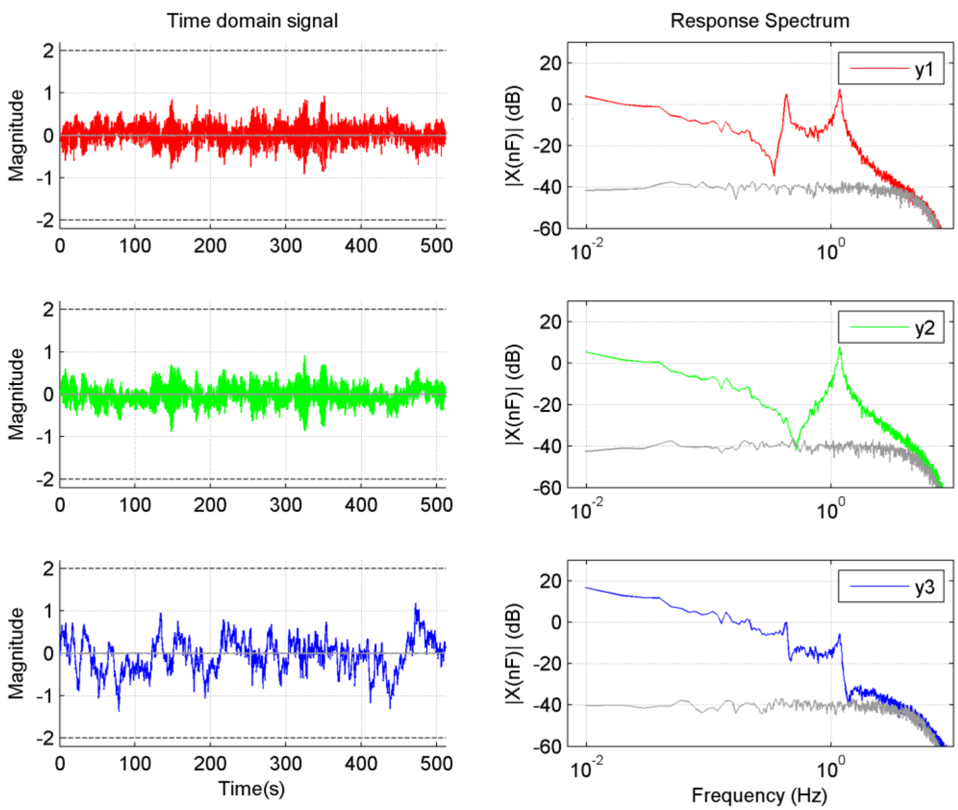


**Figure 3 time domain signal and spectrum of the sensor response for u2 activated**

## Task 1e. Frequency Response Estimation [12 pts]

Frequency response H1 estimates were generated by calculating the cross spectrum of the output with the input divided by the auto-spectrum of the input. This cross and auto spectrum were generated using the cpsd() function. The coherence was calculated too which represents the division of the two cross spectrum over the two auto spectrum. Below are figure 4 to figure 9 that display the Bode diagram of the frequency response and the coherence for each path.

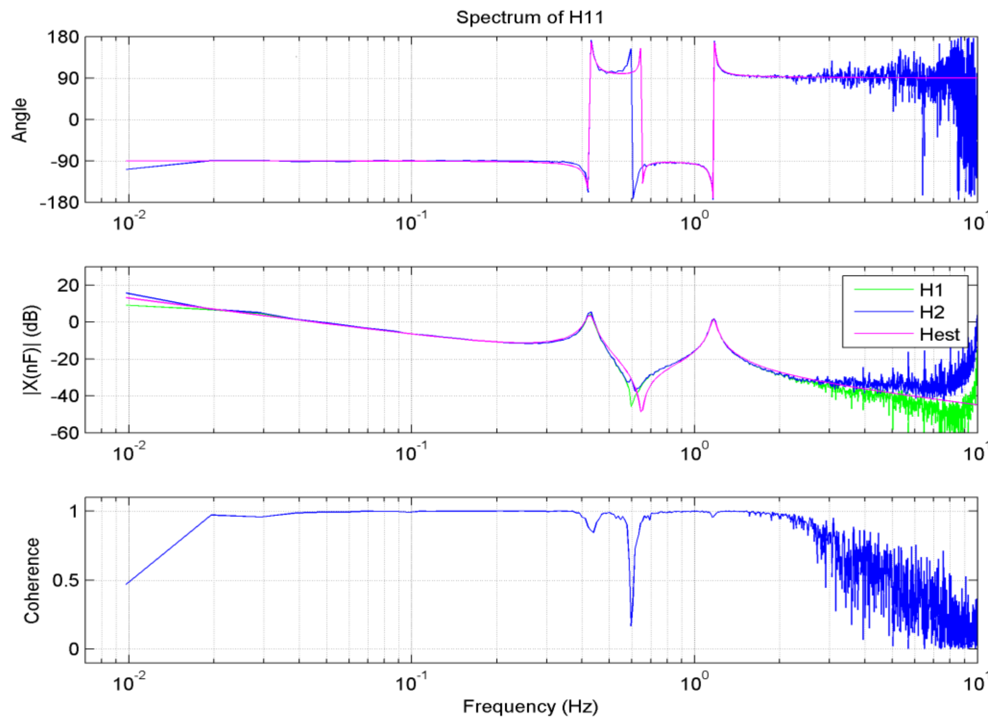


Figure 4 Frequency response and coherence of H11

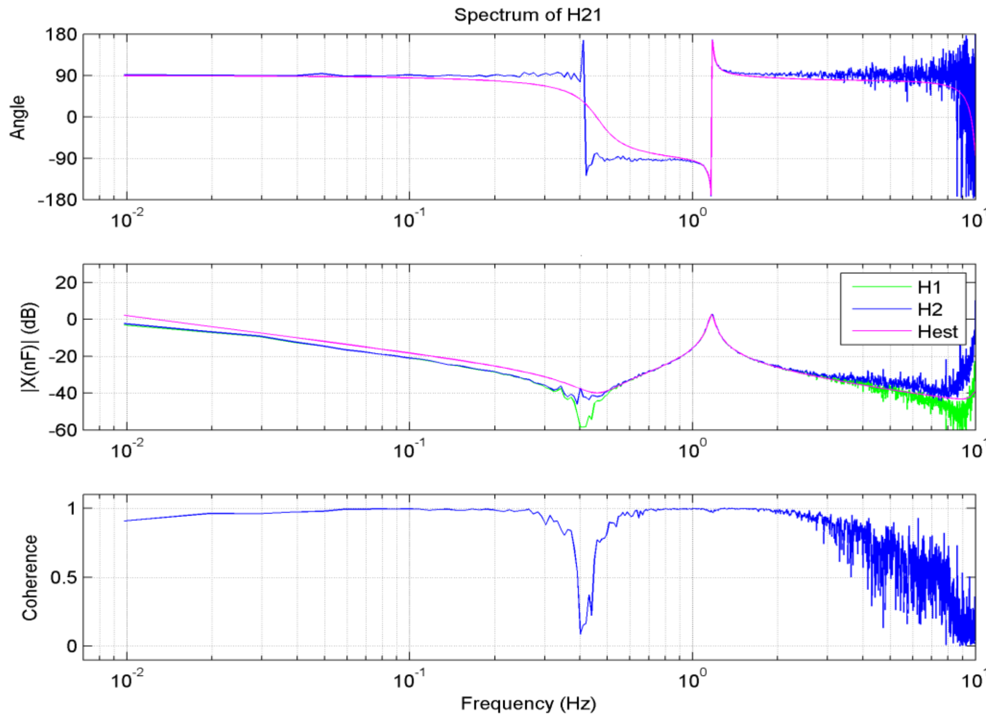


**Figure 5 Frequency response and coherence of H21**

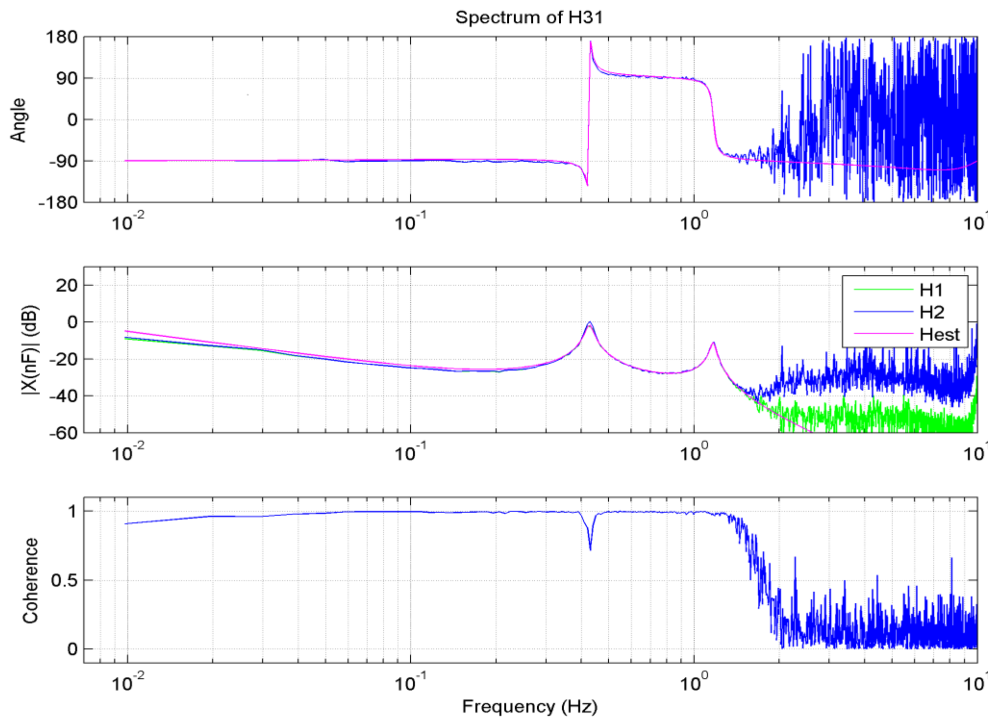


**Figure 6 Frequency response and coherence of H31**

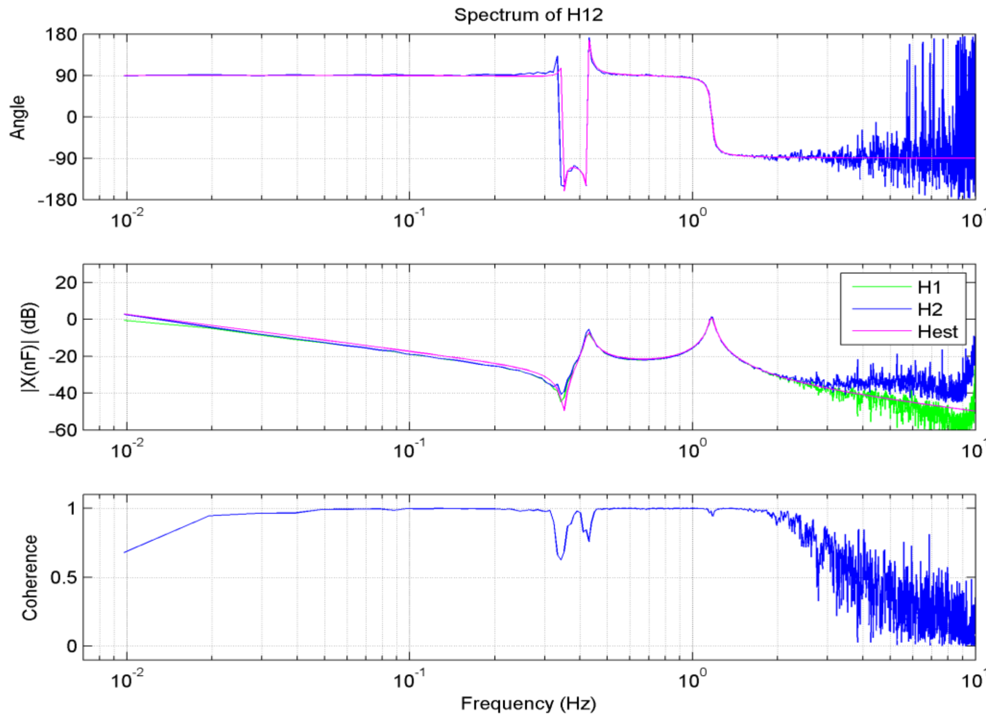


**Figure 7 Frequency response and coherence of H112**

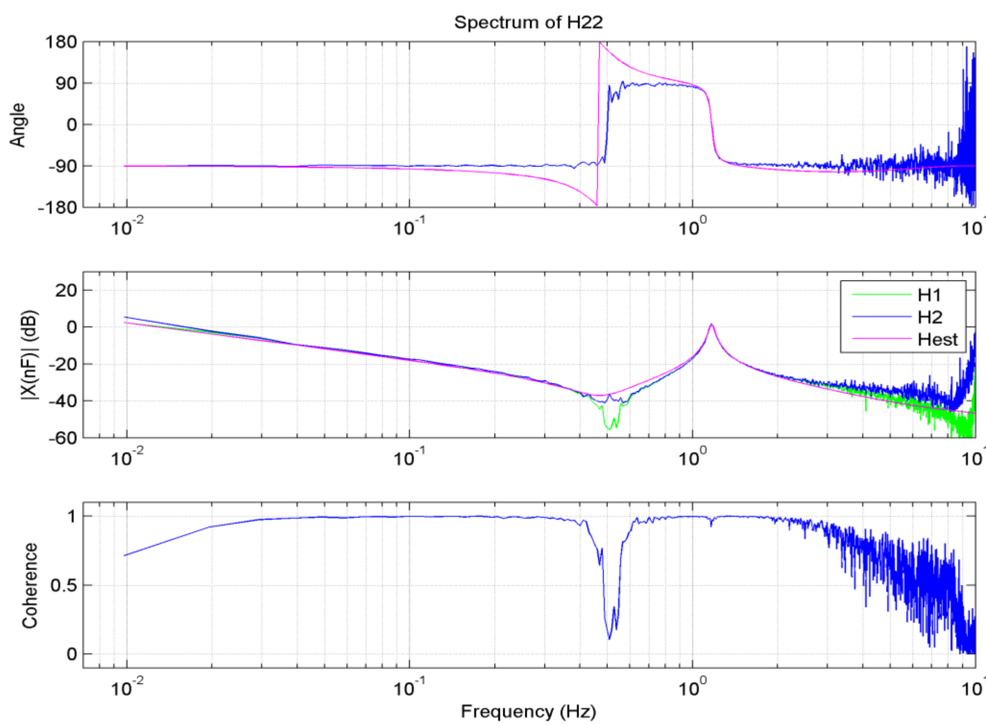


**Figure 8 Frequency response and coherence of H21**

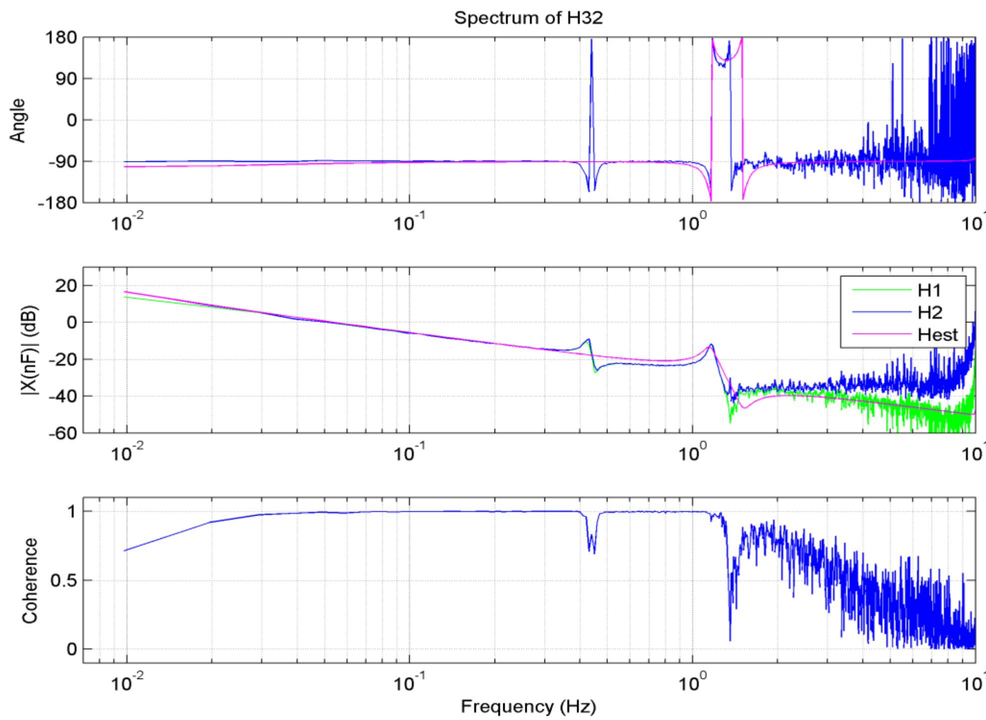


**Figure 9 Frequency response and coherence of H31**

## Task 1f. Discrete-Time Transfer Function Models [8 pts]

To determine the discrete time transfer function for each model we divided our frequency response H1 estimate by the frequency response of an integer order 1. This was used to reduce the number of pole that the function INVFREQZ() has to locate. In fact we could see from our frequency response estimate a slope of -20dB per decade at low frequencies, thus, help us to locate our first pole at z=0. Most of the frequency response estimate show two resonance, or flexible mode, thus we could estimate two couple of complex poles for the system, thus the system should have a minimum of 5 poles.
Having each H estimate for each path and the number of pole we could use the previous function to find the discrete time transfer function of each path. We set the number of zeros equal to the number of pole, then we reduce this number if the transfer function estimated did not match the corresponding H estimate. It is important to save the data from the experimentation and then apply the finding transfer function method on this data. To obtain precise result, it is important also to obtain first the best SNR.
We estimated also the number of pole and zeros by identifying the number of free integer L, the Relative degree R and the variation of the slope for each curve. We could

see then that the fact that the first mode does not appear on few graph was due to a number of zeros higher in order at the location of this mode.

$$H11 = \frac{-0.3248\, z^4 + 1.302\, z^3 - 1.954\, z^2 + 1.302\, z - 0.3244}{z^5 - 3.828\, z^4 + 5.636\, z^3 - 3.784\, z^2 + 0.975\, z}$$

Number of zeros=4 Number of poles=5

$$H21 = \frac{-0.3165\, z^4 + 0.3585\, z^3 + 0.2776 z^2 - 0.3664\, z + 0.03138}{z^5 - 1.287\, z^4 - 0.4974\, z^3 + 1.362\, z^2 - 0.4279\, z}$$

Number of zeros=4 Number of poles=5

$$H31 = \frac{0.05719\, z^4 - 0.2314\, z^3 + 0.3512\, z^2 - 0.2366\, z + 0.05987}{z^5 - 3.828\, z^4 + 5.637\, z^3 - 3.784\, z^2 + 0.9776\, z}$$

Number of zeros=4 Number of poles=5

$$H12 = \frac{0.1909\, z^4 - 0.7771\, z^3 + 1.171\, z^2 - 0.7749 z + 0.1898}{z^5 - 3.828\, z^4 + 5.637\, z^3 - 3.784\, z^2 + 0.9776\, z}$$

Number of zeros=4 Number of poles=5

$$H22 = \frac{0.3629\, z^4 - 1.349\, z^3 + 1.873\, z^2 - 1.148 z + 0.2611}{z^5 - 3.629 z^4 + 5.051\, z^3 - 3.186\, z^2 - 0.164\, z}$$

Number of zeros=4 Number of poles=5

$$H32 = \frac{0.03135\, z^4 + 0.1209\, z^3 + 0.4937\, z^2 + 0.5387 z - 0.1972}{z^5 - 2.888 z^4 + 2.936\, z^3 - 1.082\, z^2 + 0.03354\, z}$$

Number of zeros=4 Number of poles=5

## Task 1g. Discrete-Time Estimated Pole Locations [4 pts]

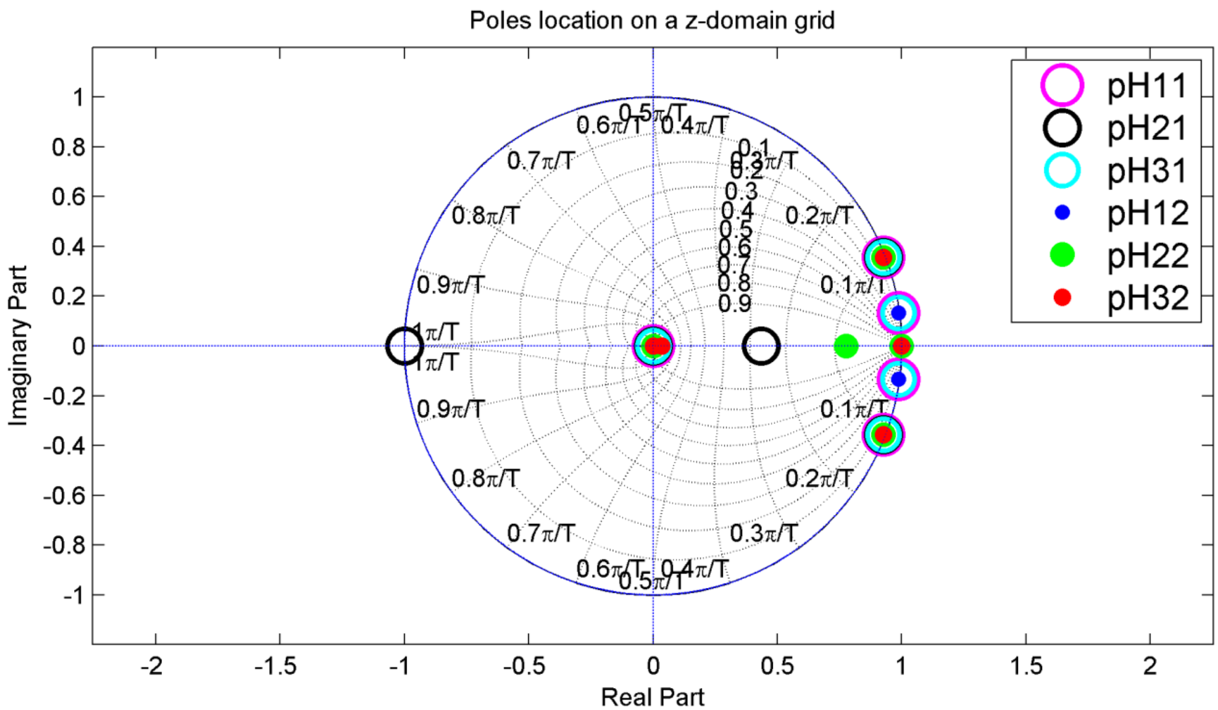


Figure 10 Localization of the poles of the plant system

## Task 1h. Discrete-Time Estimated Pole Locations [10 pts]

Figure 10 shows the localization of the poles of the system. We saw in the given equations that 3 equations have the same denominator. In fact, their poles are located proximately at the same place ( pH31, pH12, pH11), thus defined as cluster. The fact that there are zeros with higher order than the first pole mode leads pH21 having its estimated pole at fs. It was difficult for the function to estimate the first mode pole since it was cancel out by zeros. Same for pH22, one of its pole is located at 0 frequency or DC . For pH32 the pole and zeros are not at the same location but really close that leads the simulator to miss the right location of the pole.

Table1: Identification of the cluster poles

| S1/A1 | S2/A1 | S3/A1 | S1/A2 | S2/A2 | S3/A2 |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0.9264 +0.3552i | 0.9263 +0.3554i | 0.9265+ 0.3554i | 0.9264 + 0.3555i | 0.9263 + 0.3556i | 0.9268 + 0.3556i |
| 0.9264 - 0.3552i | 0.9263 - 0.3554i | 0.9874 + 0.1329i | 0.9264 - 0.3555i | 0.9263 - 0.3556i | 0.9268 - 0.3556i |
| 0.9875 +0.1335i | | 0.9874 + 0.1329i | 0.9875 +0.1335i | | |
| 0.9875 - 0.1335i | | 0.9874 - 0.1329i | 0.9875 - 0.1335i | | |

### Task 1i. Flexible Mode Damping and Natural Frequency [6 pts]

Table 2: Flexible Mode Damping and Natural Frequency of the two flexible modes

| Estimated Complex Pole | $f_n$ (Hz) | zeta | Description |
|---|---|---|---|
| 0.9264 +0.3552i | 0.4265 | 3.02e-02 | Flexible mode |
| 0.9875 +0.1335i | 1.1666 | 2.16e-02 | Flexible mode |

### Task 1j. Discrete-Time State-Space Model [6 pts]

The plant is a MIMO system with two inputs and outputs so 6 paths. We identified the system to have 5 poles. These poles should be the same for each path but our results show that there were sometime really close or completely different. Due to this difference, even small, our simulator estimated a number of state to be 30 for the State space Model, equal of the number of poles time number of path.

Thus the dimension of the matrices A,B,C and D of the model are

A:30∗30 B:30∗2 (nbr of state , nbr of output)

C: 3∗30 (nbr of intput ,nbr of state) D: 3∗2(nbr of intput , nbr of output)

We can reduce the number of state to 5 if we set the same right pole for each path. This step will be processing in another study.

To create the State Space Model, I created a 3*2 matrix of transfer function of each path and inserted in the SS() function that provide a LTI object of the system.

### Task 1k. FRF of Discrete-Time State-Space Model [10 pts]

- Please refer on Task 1 E for the requested plot.
- There were a tread off between choosing the number of pole of 5 or 6. We decided to have a number of pole of 5 because they gave use the necessary information of the model which were the localization of the two flexible mode and the fixed mode ( the integration at low frequency). Added a higher order of pole made for few path a better curve shipping and for other there were no real effect. We can also go higher in the order of pole, but these adding pole in the modeling system would be only of order one which do really no affect the dynamic model ( no resonance).
- Observing the plots, we can state that our result are pretty accurate the discrete state space estimate match the H estimate generated experimentally for each path. For a better approximation, it would be necessary to add force a set of pole for every path. This set of poles would be the one defined as the closest. Therefore, The simulator will not have to find the localization of the pole but only place the zeros.